

Mathematics II

FINAL EXAMINATION STUDY & PRACTICE GUIDE

BEng. Degree in Industrial Systems

Component	Weight	Duration	Details
Continuous Assessment	50%	Throughout semester	Quizzes, assignments, mid-term
Final Examination	50%	3 hours (likely)	Covers all 3 units

Final Exam Typical Structure:

- Section A: Short answer / formula recall (20–30%)
- Section B: Problem-solving (50–60%)
- Section C: Application / extended response (15–20%)

Topic Weighting (Estimated)

Unit	Topic	Approx. Exam Weight
1	Sequences & Series (AP/GP)	15%
2	Differential Calculus	50%
3	Integral Calculus & Applications	35%

Within Differential Calculus (Unit 2), focus on:

- Derivatives of standard functions (algebraic, trig, hyperbolic, exp, log)
- Implicit & parametric differentiation
- Partial derivatives & maxima/minima/saddle points
- Taylor & McLaurin series

Within Integral Calculus (Unit 3), focus on:

- Definite/indefinite integrals (substitution, by parts)
- Area under curve, volume of revolution, centroid
- Numerical integration (trapezoidal, Simpson's, mid-ordinate)

Self-Assessment Checklist

Before the exam, tick off each skill:

Unit 1: Sequences & Series

- Distinguish between sequence and series
- Find n th term of AP and GP
- Find sum of first n terms of AP and GP
- Solve word problems (population, loans, etc.)

Unit 2: Differential Calculus

- Compute limits of functions
- Differentiate from first principles (polynomial, $\sin x$, $\cos x$)
- Differentiate algebraic, trig, hyperbolic, exp, log functions
- Apply product, quotient, chain rules
- Differentiate implicit functions
- Differentiate parametric functions
- Find second and higher derivatives
- Apply Leibniz's theorem
- Compute partial first and second derivatives
- Find maxima, minima, saddle points for $f(x,y)$
- Expand functions using McLaurin's series
- Expand functions using Taylor's series
- Use series for approximation

Unit 3: Integral Calculus

- Evaluate indefinite integrals (standard forms)
- Use substitution method
- Use integration by parts
- Evaluate definite integrals
- Find area under a curve
- Compute volume of revolution (x-axis, y-axis)
- Find centroid by integration
- Apply trapezoidal rule
- Apply Simpson's rule
- Apply mid-ordinate rule



Sample Examination Questions

Section A: Short Answer (Formula Recall)

- A1.** Write the formula for the n th term of a Geometric Progression.
A2. State Leibniz's theorem for the n th derivative of a product.
A3. Write the first three non-zero terms of the McLaurin series for $\sin x$.
A4. State the condition for a saddle point of $f(x, y)$.
A5. Write the trapezoidal rule formula for $\int_a^b f(x)dx$ with n strips.
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Section B: Problem Solving

B1. Sequences & Series

The 3rd term of an AP is 12 and the 7th term is 28. Find:

- (a) First term and common difference
(b) Sum of the first 10 terms

B2. Differentiation from First Principles

Differentiate $f(x) = x^3$ from first principles.

B3. Derivatives of Standard Functions

Find $\frac{dy}{dx}$ for:

- (a) $y = e^{2x} \sin(3x)$
(b) $y = \ln(\cosh x)$
(c) $y = \tan^{-1}(x^2)$

B4. Implicit Differentiation

If $x^2y + \sin y = 3x$, find $\frac{dy}{dx}$.

B5. Partial Derivatives & Extremum

Given $f(x, y) = x^3 - 3x + y^2 - 4y$, find:

- (a) First partial derivatives f_x, f_y
(b) Critical points
(c) Classify each as max, min, or saddle point

B6. McLaurin Series

Derive the McLaurin series for $\cos x$ up to the x^4 term.

B7. Integration by Substitution

Evaluate $\int \frac{2x}{1+x^2} dx$.

B8. Integration by Parts

Evaluate $\int xe^{2x} dx$.

B9. Definite Integral & Area

Find the area bounded by $y = x^2$ and the x-axis from $x = 0$ to $x = 2$.

B10. Volume of Revolution

The curve $y = \sqrt{x}$ from $x = 0$ to $x = 4$ is rotated about the x-axis. Find the volume generated.

B11. Numerical Integration (Trapezoidal Rule)

Approximate $\int_0^2 e^{x^2} dx$ using the trapezoidal rule with 4 strips ($n=4$). Use $e^0 = 1$, $e^{0.25} = 1.284$, $e^1 = 2.718$, $e^{2.25} = 9.488$, $e^4 = 54.598$. (Values approximated for calculation)

Section C: Extended Response/Application

C1. Taylor Series Approximation

Expand $\ln(1+x)$ as a Taylor series about $x = 0$ up to the x^3 term. Use it to approximate $\ln(1.1)$ and compare with the true value (0.09531).

C2. Partial Derivatives & Small Changes

The volume of a cylinder is $V = \pi r^2 h$. If $r = 5$ cm, $h = 10$ cm, and the possible errors are $\Delta r = 0.1$ cm and $\Delta h = 0.2$ cm, estimate the maximum error in V .

C3. Centroid by Integration

Find the centroid of the area under $y = x^2$ from $x = 0$ to $x = 2$. (Formulas: $\bar{x} =$

$$\frac{\int xy \, dx}{\int y \, dx}, \bar{y} = \frac{\int \frac{1}{2}y^2 \, dx}{\int y \, dx}$$

C4. Numerical Integration Comparison

Approximate $\int_0^1 \frac{4}{1+x^2} dx$ (which equals π) using:

- (a) Trapezoidal rule with $n=4$
- (b) Simpson's rule with $n=4$
- (c) Mid-ordinate rule with $n=4$

Compare errors.

Answers & Hints

A1. $a_n = a_1 r^{n-1}$

A2. $(uv)^{(n)} = \sum_{k=0}^n \binom{n}{k} u^{(k)} v^{(n-k)}$

A3. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

A4. $D = f_{xx}f_{yy} - (f_{xy})^2 < 0$

A5. $\int_a^b f(x) dx \approx \frac{h}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)], h = \frac{b-a}{n}$

B1. (a) $a = 4, d = 4$ (b) $S_{10} = 220$

B2. $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = 3x^2$

B3. (a) $2e^{2x} \sin(3x) + 3e^{2x} \cos(3x)$ (b) $\tanh x$ (c) $\frac{2x}{1+x^4}$

B4. $\frac{dy}{dx} = \frac{3-2xy}{x^2+\cos y}$

B5. (a) $f_x = 3x^2 - 3, f_y = 2y - 4$ (b) Critical points: (1,2), (-1,2) (c) (1,2) \rightarrow min, (-1,2) \rightarrow saddle

B6. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

B7. $\ln(1+x^2) + C$

B8. $\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

B9. $\frac{8}{3}$ square units

B10. 8π cubic units

B11. Approximation $\approx \frac{0.5}{2} [1 + 2(1.284 + 2.718 + 9.488) + 54.598] =$ check calculation

C1. $\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3}$. For $x = 0.1$: $0.1 - 0.005 + 0.000333 = 0.095333$ (error ~ 0.00002)

C2. $\Delta V \approx \frac{\partial V}{\partial r} \Delta r + \frac{\partial V}{\partial h} \Delta h = 2\pi r h \Delta r + \pi r^2 \Delta h = 2\pi(5)(10)(0.1) + \pi(25)(0.2) = 10\pi + 5\pi = 15\pi \approx 47.12 \text{ cm}^3$

C3. $\bar{x} = \frac{\int_0^2 x \cdot x^2 dx}{\int_0^2 x^2 dx} = \frac{16/4}{8/3} = \frac{4}{8/3} = 1.5, \bar{y} = \frac{\int_0^2 \frac{1}{2} x^4 dx}{8/3} = \frac{16/10}{8/3} = \frac{1.6}{2.666} = 0.6$

C4. Exact value $= \pi \approx 3.14159$. Simpson's rule will be most accurate.



Final Exam Success Strategies

2 Weeks Before Exam

- Complete all practice questions above
- Make a **one-page formula sheet** (even if not allowed, it helps memory)
- Identify weak topics → rewatch lectures/read textbook

1 Week Before Exam

- Take a **3-hour mock exam** (create from sample Qs)
- Practice **without looking at notes**
- Review error log

Day Before Exam

- Light review of formulas
- Sleep early (7+ hours)

During Exam

- **Read all questions first** – attempt easiest ones
- Show all steps (partial credit matters)
- For series expansions, write the general term
- Check: did I forget $+C$ for indefinite integrals?
- For partial derivatives, verify $f_{xy} = f_{yx}$



Mistake	Correction
Forgetting chain rule in e^{3x^2}	Derivative = $6xe^{3x^2}$
Mixing AP and GP formulas	AP: linear, GP: exponential
Wrong substitution choice	Try $u =$ inner function first
Volume formula missing π	$V = \pi \int [f(x)]^2 dx$
Simpson's rule odd number of strips	Requires even number of strips
Saddle point classified as extremum	Check $D < 0 \rightarrow$ saddle

LAST-MINUTE FORMULA SUMMARY

AP: $a_n = a + (n - 1)d$, $S_n = \frac{n}{2}(2a + (n - 1)d)$

GP: $a_n = ar^{n-1}$, $S_n = a \frac{1-r^n}{1-r}$

Derivative (first principles): $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Product rule: $(uv)' = u'v + uv'$

Leibniz: $(uv)^{(n)} = \sum_{k=0}^n \binom{n}{k} u^{(k)} v^{(n-k)}$

Partial derivative test: $D = f_{xx}f_{yy} - (f_{xy})^2$

McLaurin: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

Taylor: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$

Integration by parts: $\int u dv = uv - \int v du$

Volume (x-axis): $V = \pi \int_a^b [f(x)]^2 dx$


Trapezoidal rule: $\frac{h}{2} [f_0 + 2f_1 + \dots + 2f_{n-1} + f_n]$


Simpson's rule: $\frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + f_n]$ (n even)


Good luck on your final examination! Practice daily, stay confident, and show all your working. 🎓


ADDITIONAL SUPPORT

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